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Reduction and optimisation of thermal models using Kirchhoff network theory

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Abstract

The network topology reduction is based on an exact mathematical tool. The principle is a reduction of the problem matrix to a few lines and columns. The original reduction method developed in this study is an extension of this technique implemented to transient linear heat diffusion problems. The finite volume model is hence reduced to a few chosen nodes and a new type of element appears, the compensation capacitance. This element, discussed in [Lagonotte et al., European Phys. J. Appl. Phys. 13 (3) (2001) 177], links every node of the model and improves the dynamic behaviour of the model. Two steps of optimisation, concerning the choice of the nodes and the value of the compensation capacitors, are discussed. The method is implemented to an industrial object, a 5.5 kW squirrel cage induction machine, which model is reduced from 1140 down to 10 nodes.

Keywords: Thermal modelling; Model reduction; Nodal method; Network theory; Electric machine

1. Introduction

The use of reduced models is criticised nowadays because of fast progresses in computing science, constantly reducing the computing times. However, some implementations still need very simplified accurate models as real-time surveillance of an industrial object thermal behaviour, for instance. Experimental studies using inverse techniques will also need reduced models. A better reliability in transient regime of such models is still in demand, as well as an easy implementation of the reduction method itself.

Most reduction methods in literature are based on the truncation of the state representation (1) of a heat diffusion problem through a change of base, thus obtaining a new representation with a much lower order (2).

$$\begin{cases} \dot{T}(t) = \boldsymbol{A}T(t) + \boldsymbol{B}U(t) \\ Y(t) = \boldsymbol{C}T(t) \end{cases} \tag{1}$$

$$\begin{cases} \dot{X}(t) = \boldsymbol{A}_r X(t) + \boldsymbol{B}_r U(t) \\ \widehat{Y}(t) = \boldsymbol{C}_r X(t) + \boldsymbol{D}_r U(t) \end{cases} \tag{2}$$

The Moore method [1] consists on analysing the abilities of control and observation to build the transfer matrix used for a change of base. The input influence on the state X and the weight of each component of X on the output vector Y are studied. The system of equations is truncated in this new base where controlability and observability are the best. The Eitelberg method [2] consists on keeping some chosen components of the temperature vector T. The reduced state matrix is then identified by the minimization of a quadratic criterion when both reduced and detailed model are submited to the same input (step or Dirac).

The model representation, built as a coupled equations system, can also be diagonalised to an uncoupled equations system. The influence of the various modes on the system thermal behaviour is then studied and the new reduced representation is deduced by selecting a combination of the modes through various choice criteria. These are the modal methods from Marshall's method [3] keeping the largest time constants to the agregation method [4] using an energetic interpretation of the modes. The obtained model can then be optimised by minimization of a criterion [5]. The reduced model structure can also be postulated and its parameters identified from experiments. In these cases (except for Eitelberg method), we have a mathematical

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Nomenclature

$\boldsymbol{A}$	state matrix (dim .N, N)	T_{d}	detailed model results ..... K
$\boldsymbol{B}$	input (or control) matrix (dim .N, p)	T_{r}	reduced model results ..... K
$\boldsymbol{C}$	observation matrix (dim .q, N)	U	input vector
C	thermal capacitance ..... $\mathrm{J{\cdot}K^{-1}}$	$\boldsymbol{V}$	boundary conditions vector
$\boldsymbol{C}_{\mathrm{a}}$	thermal capacitance matrix	X	state vector in the reduced model base
$\boldsymbol{G}$	conductance matrix	$\boldsymbol{Y}$	admittance matrix
j	unitary imaginary number	Y	output vector of the detailed model
J	error criterion	$\widehat{Y}$	output vector of the reduced model
N_{o}	number of observation nodes	$\boldsymbol{Z}$	impedance matrix
N_{s}	number of supplementary nodes	*Greek symbols*	
s	Laplace variable ..... $\mathrm{s^{-1}}$	ε	absolute error
$\boldsymbol{P}$	source flux matrix	ϕ	heat flux ..... W
$\boldsymbol{R}$	resistance matrix	Φ	flux Laplace transform
t_{s}	simulation time ..... s	θ	Heating Laplace transform
T	temperatures vector (state vector)	ω	pulsation ..... $\mathrm{rad{\cdot}s^{-1}}$
T_{a}	ambient temperature ..... K		

model, the elements of which can hardly be physically interpreted.

The reduction method described hereafter is based on the electric network theory (see [6,7]). The method principle [8] rests on the physical interpretation of the conductance matrix $\boldsymbol{G}$ and its inverse, the resistance matrix $\boldsymbol{R}$. The matrices $\boldsymbol{G}$ and $\boldsymbol{R}$ only lead to the stationary state of the system. Therefore, we introduce the admittance and impedance matrices $\boldsymbol{Y}$ and $\boldsymbol{Z}$, containing the model structure in the Laplace space.

We observe in the reduced model the presence of a supplementary element, a capacitance of negative value, discussed in the frame of nodal method improvement in reference [9]. This element, which we call compensation capacitance, makes a contribution in the dynamic response of the model.

The reduction is discussed with the help of the simple case of a wall and then completed by an optimisation of the choice of the nodes and by the optimisation of the compensation capacitances values. An implementation to a complex industrial system is then presented dealing with an electric induction motor. The reduced model results, compared to the detailed model's ones, are eventually presented.

2. Definition of the problem

Let us consider a thermal problem treated by the nodal method. This method's principle is to mesh the object as elementary volumes assumed isothermal. An energy balance written on each of these volumes, the actual nodes, leads to a coupled algebraic-differential equations system. The coupling taken into account between each nodes can model a transfer by conduction but also by convection, radiation or fluid transport through a linearised conductance. The herein presented reduction method is limited to linear problems.

One can express the obtained equations system under the state representation form:

$$\dot{T} = -\boldsymbol{C}_{\mathrm{a}}^{-1}\boldsymbol{G}\cdot T + \boldsymbol{C}_{\mathrm{a}}^{-1}\boldsymbol{P} \tag{3}$$

2.1. The conductance matrix $\boldsymbol{G}$

The conductance matrix $\boldsymbol{G}$ is very easily written, even for important meshed networks. It has the following particularities:

- According to our agreement, all the diagonal terms are positive.
- Non-diagonal terms are all negative or null, thus satisfying Kirchhoff's law.
- The matrix is symmetrical ($G_{ij} = G_{ji}$) except if there is a fluid transport conductance, for in that case a node's temperature is only dependant of the upstream node(s) (unilateral conductance).
- In general, most of non-diagonal terms are null, according to the fact that each node has its energy balance only linked to its direct neighbourhood. The matrix is sparse, interactions are local ones.
- The system topology can be derived from $\boldsymbol{G}$.

2.2. The resistance matrix $\boldsymbol{R}$

The matrix $\boldsymbol{R}$ is calculated inverting $\boldsymbol{G}$. It is a full matrix for if we inject a flux in the node i, we obtain the resulted temperature rise in every node of the model as a function of each term of the matrix, in static regime: $\boldsymbol{\Delta T} = \boldsymbol{R}\cdot\boldsymbol{\phi}$ that is:

$$\Delta T_1 = R_{1i}\cdot\phi_i, \qquad \Delta T_2 = R_{2i}\cdot\phi_i, \ldots$$
$$\Delta T_i = R_{ii}\cdot\phi_i, \ldots$$

$$\Delta T_{n-1} = R_{n-1\,i} \cdot \phi_i, \qquad \Delta T_n = R_{ni} \cdot \phi_i$$

One obtains the static regime image of the heat diffusion in the whole system caused by an injection in i. $\boldsymbol{R}$ is therefore a sensitivity matrix.

The diagonal term R_{ii} corresponds to the equivalent resistance of the whole network seen from the considered node, $\Delta T_i = R_{ii} \cdot \phi_i$ (input impedance between i and the reference node). Non-diagonal terms an provide the mutual influence between every couple of node. Physically, a flux injection ϕ_j will generate temperature variations everywhere in the network and in particular, in the node j, T_j will vary proportionally to R_{ij}.

One can summarise the principal characteristics of the matrix $\boldsymbol{R}$:

- The matrix $\boldsymbol{R}$ is symmetrical ($R_{ij} = R_{ji}$) unless in the presence of unilateral conductances in the network.
- The matrix $\boldsymbol{R}$ is mathematically full. It contains intrinsically the physical interactions between every couple of nodes, leaving the advantage to give a global vision of the temperature diffusion in all the network caused by an injection of flux in a single node.
- The matrix $\boldsymbol{R}$ intrinsically contains static regime reduced models (obtained by Gauss elimination).

Eventually, as the matrix $\boldsymbol{G}$ and $\boldsymbol{R}$ are inverse one from the other, it contain intrinsically the same information. These equivalent informations are presented under two different forms, locally for the matrix $\boldsymbol{G}$ and globally for $\boldsymbol{R}$.

2.3. Reformulation of the problem in the Laplace space

So as to turn to a reduced model we introduce another formulation, derived from the Laplace transform, and using complex matrices for the numerical computation. First of all, temperatures are expressed as temperature rises compared to the ambient temperature, considered continuously as the reference temperature. Then, we apply the Laplace transform to the system.

The transformed Eq. (3) can then be expressed as:

$$\boldsymbol{C}_{\mathrm{a}} \cdot s \cdot \boldsymbol{\theta} = -\boldsymbol{G} \cdot \boldsymbol{\theta} + \boldsymbol{\Phi} \tag{4}$$

Therefore we define the *admittance matrix* by $\boldsymbol{Y}(\boldsymbol{s}) = (\boldsymbol{G} + \boldsymbol{C}_{\mathrm{a}} \cdot s)$, containing the conductance network as well as the thermal capacitances, which leads us to the new problem formulation:

$$\boldsymbol{\Phi} = \boldsymbol{Y}(\boldsymbol{s}) \cdot \boldsymbol{\theta} \tag{5}$$

The inverse of the admittance matrix is then called the *impedance matrix* and noted $\boldsymbol{Z}(\boldsymbol{s})$. So we assembled in a single matrix all the information concerning the considered network structure. This matrix contains both the static and dynamic characteristics, the reduction operation is applied to it.

3. The model reduction method

The network reduction method proceeds by the selection of nodes to keep among those of the detailed model. The obtained reduced model keeps a structure analogue to the detailed one, which can be represented as an equivalent electric network, and linking the few chosen nodes together. The heat input flux conditions are then automatically distributed among those nodes and only their temperature will be accessible in simulation.

3.1. Reduction of the network

The reduction process starts with the separation of the system in two sub-systems. We therefore consider at this stage that a choice has been done and that only a few nodes among the detailed model's ones will be kept. This stage of the choice of the nodes constitutes one of the main part of the method and is described in Section 4.1. Thus, the problem is expressed as:

$$\begin{bmatrix} \boldsymbol{\Phi_1} \\ \boldsymbol{\Phi_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y_{11}}(s) & \boldsymbol{Y_{12}}(s) \\ \boldsymbol{Y_{21}}(s) & \boldsymbol{Y_{22}}(s) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\theta_1} \\ \boldsymbol{\theta_2} \end{bmatrix} \tag{6}$$

The index 1 concerns the kept nodes and the index 2 the eliminated ones. Defining $\boldsymbol{Z}$ as the model's impedance matrix which is the inverse of $\boldsymbol{Y}$, the formulation of θ_1 from the system (6) leads us to:

$$\begin{cases} \boldsymbol{\theta_1} = \boldsymbol{Z_{11}} \cdot \left(\boldsymbol{\Phi_1} - \boldsymbol{Y_{12}} \cdot \boldsymbol{Y_{22}^{-1}} \cdot \boldsymbol{\Phi_2}\right) \\ \boldsymbol{Z_{11}} = \left(\boldsymbol{Y_{11}} - \boldsymbol{Y_{12}} \cdot \boldsymbol{Y_{22}^{-1}} \cdot \boldsymbol{Y_{21}}\right)^{-1} \end{cases} \tag{7}$$

Thus, if we define the reduced matrices $\boldsymbol{Y}_{\mathrm{red}}$ and $\boldsymbol{\Phi}_{\mathrm{red}}$ as

$$\begin{cases} \boldsymbol{Y}_{\mathrm{red}} = \boldsymbol{Y_{11}} - \boldsymbol{Y_{12}} \cdot \boldsymbol{Y_{22}^{-1}} \cdot \boldsymbol{Y_{21}} = \boldsymbol{Z_{11}^{-1}} \\ \boldsymbol{\Phi}_{\mathrm{red}} = \boldsymbol{\Phi_1} - \boldsymbol{Y_{12}} \cdot \boldsymbol{Y_{22}^{-1}} \cdot \boldsymbol{\Phi_2} \\ \qquad\quad = \boldsymbol{\Phi_1} + \boldsymbol{Z_{11}^{-1}} \cdot \boldsymbol{Z_{12}} \cdot \boldsymbol{\Phi_2} \end{cases} \tag{8}$$

we can now express the problem as a reduced system containing as much equations as kept nodes:

$$\boldsymbol{\Phi}_{\mathrm{red}} = \boldsymbol{Y}_{\mathrm{red}}(s) \cdot \boldsymbol{\theta_1} \tag{9}$$

The first expression of (8) shows that one can obtain from the complete admittance matrix the reduced matrix of the admittances containing the actual reduced model. The network reduction operation can then be carried out according to the diagram of Fig. 1. The matrix $\boldsymbol{Y}$ (from expression (6)) is inverted. The sub-matrix $\boldsymbol{Z_{11}}$ corresponding to the nodes to be kept is extracted from the obtained matrix. A new inversion is applied producing the admittance matrix of the reduced network $\boldsymbol{Y}_{\mathrm{red}}$. One can notice that the inversion, computing time consuming, is carried out once for all, whichever nodes are chosen for the reduced model. One can thus extract from the impedance matrix as much reduced networks as desired in a simple inverting operation on a small sized matrix.

If we could invert literally, we would obtain a reduced model analytically exact expressed as a function of s. However, the literal matrix inversion cannot be applied to

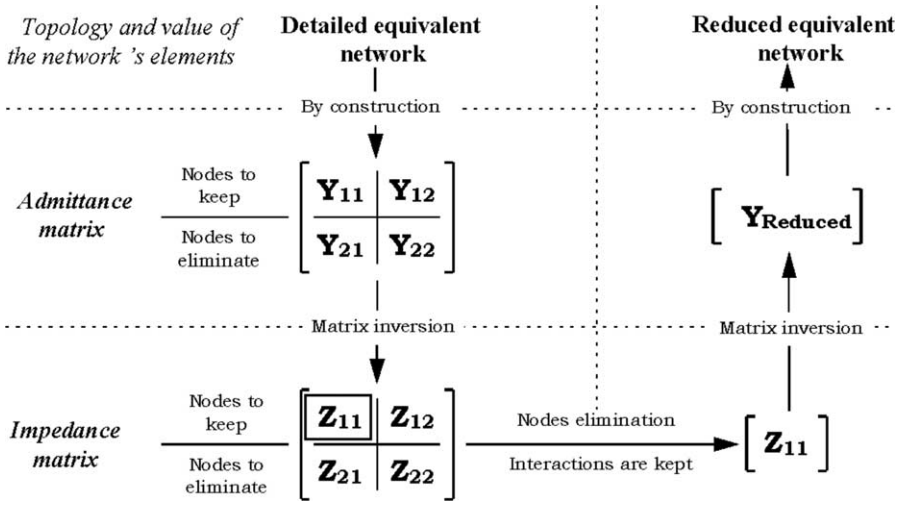


Fig. 1. Methodology for the reduction of a dynamic system's physical model.

large systems and numerical inversion will require a new formulation of the problem as complex matrices. To do so, we substitute the Laplace variable s by the imaginary number $j\omega$.

The complex admittance matrix $\boldsymbol{Y}(\omega)$ is then composed by a *conductance* as the real part and by a *susceptance* as the imaginary part. Then, for the numerical determination of the reduced admittance matrix $\boldsymbol{Y}_{\text{red}}(\omega)$, we need to make a first approximation by choosing a reference pulsation value ω_{r} before we apply the method described in Fig. 1.

If we replace $\boldsymbol{Y}$ by the real matrix of conductances $\boldsymbol{G}$, the reduced model obtained is numerically exact to compute static regimes. In our case, the dynamic reduced model is exact for the reference pulsation ω_{r}. That is to say that for harmonic excitations with the pulsation ω_{r} (chosen for the writing of $\boldsymbol{Y}(\omega)$), the responses at the reduced model's nodes will have exactly the same phase and amplitude than at the detailed model's corresponding nodes. Concerning the implementation of this method to the thermal problems we present here, we choose a very low reference pulsation ($\omega_{\text{r}} \approx 10^{-6}$ rad·s^{-1}) in order for the reduced model to be nearly exact in static regime ($\omega = 0$) but to also allow the computation of the dynamic reduced model's structure. This choice represents an approximation at this stage. Such a choice is very acceptable for most of possible implementations not involving high frequencies. However, further studies are in progress to obtain a better representation of the spectrum of frequencies, by taking into account a range of values of ω_{r} for instance.

The obtained reduced matrix $\boldsymbol{Y}_{\text{red}}$ is a matrix generally full and complex. It can be interpreted as a network of conductances and capacitances applying the same rules as for the writing of $\boldsymbol{Y}$.

Concerning $\boldsymbol{\Phi}_{\text{red}}$, this matrix is also complex despite the fact that the original matrix $\boldsymbol{\Phi}$ is real. We can notice from expression (8b) that each term of $\boldsymbol{\Phi}_{\text{red}}$ is a function of s (or $j\omega$ after the approximation). Thus, each source in each node of the reduced model could be itself represented

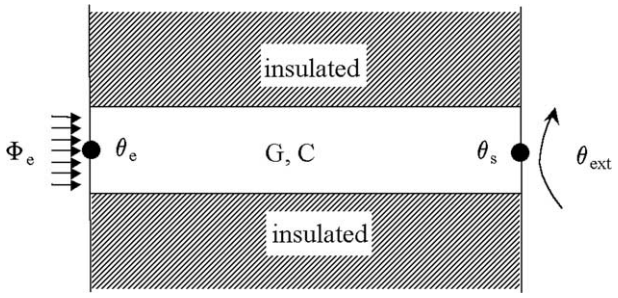


Fig. 2. Finite wall of thermal capacity C and conductance G.

as an equivalent network. If we assume that the original sources Φ_i are step functions of time, those networks would be RC filters. In that way, the heat injection in each node would not only be a damped signal but also a delayed one. This formulation would give a better representation of the distance between the real source and the reduced model's nodes. Taking this information into account would entails the addition of an important number of nodes (not representative of any physical temperature of the model), especially if different types of independant sources are involved. That is why we choose to only keep the zero order terms of this expression, which is the real part of the numerical complex value of $\boldsymbol{\Phi}_{\text{red}}$, to stay consistent with the goal of model reduction.

3.2. Structure of the reduced model

To observe the structure of the obtained reduced model, we implement the method to a model with a simple topology where just one direction of the diffusion is taken into account. Thus, a wall of finite thickness and infinite (or insulated) in the two other directions can be modelled by a succession of nodes linked by conductances and characterised by thermal capacitances.

Let us model a homogeneous wall as a series of 128 "Π-shaped" cells [10], i.e., 129 nodes linked by conduc-

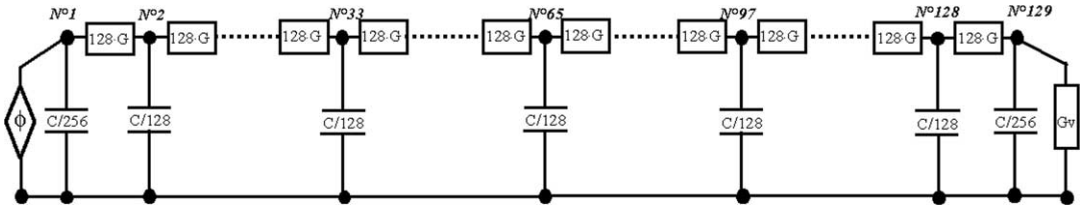


Fig. 3. 128 nodes model of the finite wall.

tances of value $g_{ij} = 128 \cdot G$ and characterised by capacitances of value $C/128$ except the two extreme nodes of which the capacitance value is $C/256$. The node 1 receives a heat flux input. The node 129 evacuates the heat by the mean of a conductance towards the reference node modelling convection on a surface in contact with ambient air. The equivalent network is represented in Fig. 3.

For numerical computation, we take values corresponding to a metallic material the thermophysical properties of which are chosen similar to iron. Then we consider a cylindrical bar of diameter 10 mm and length 128 mm perfectly insulated (cf. Fig. 2), of thermal capacity $C = 32\ \mathrm{J{\cdot}K^{-1}}$ and of which the conductance between two nodes is $6.4\ \mathrm{W{\cdot}K^{-1}}$. The conductance between the node 129 and the air is $Gv = 0.2\ \mathrm{W{\cdot}K^{-1}}$. The node 1 receives a heat flux of 10 W.

As an example, we apply the reduction to only keep the end and middle nodes (1, 65 and 129). The topology of the obtained reduced network is represented in Fig. 4. We observe that the reduced model structure contains the classical elements with the value they would have had if we had directly modelled the same wall by only three equally spaced nodes. However, to this classical nodal model are added some new elements: negative capacitances linking every node to each other in the model. These *compensation capacitances* (Cc), whose interest is studied in the reference [9], give to the reduced model the dynamic performances equivalent to a classical model of higher order. It will compensate in a way what the model have lost in dynamic due to the lowering of its order.

3.3. Highlighting degrees of freedom

Let us observe the simulation results of the reduced model of the wall presented above. The input condition is a heat flux step injected in node 1. Fig. 5 presents a comparison of the responses on nodes 1 and 129 for the detailed and reduced model. One notices that the dynamic behaviour of these responses is unsatisfying. We notably observe a strong oscillation during the first time steps for the node 129. This oscillation results from the compensation capacities to model the delay of the heat diffusion through the wall. In order to improve the reduced model's performances, one can act on two parameters.

The nodes composing the reduced model do not all have the same importance. Some will be observed by the user and the other are meant to complete it, increasing its order. The choice of these secondary nodes has an effect on the quality

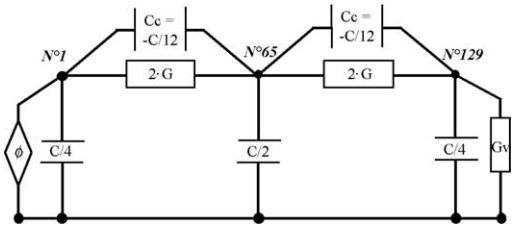


Fig. 4. Wall model reduced to the nodes 1-65-129.

of the observed nodes' response quality. One also notices that the modification of the compensation capacitances values (divided by 2 on the example Fig. 5) has a clear influence on the reduced model dynamic performances. The response in node 129 is appreciably improved but it is to the detriment of the response in node 1. These two degrees of freedom introduce the "optimisation" part of the reduced model construction.

4. Optimisation of the reduced model

The reduction operation allows to switch from a detailed model to a N_r nodes model composed by N_o nodes that are to be observed by the user and N_s supplementary nodes. These supplementary nodes allows to obtain the best compromise between quality of the dynamic response (on the observation nodes) and fastness of computation. Hence, the optimisation stage must improve as much as possible the response on the observation nodes, an optimal quality of the response on the supplementary nodes being not necessary.

4.1. Optimisation of the choice of nodes

The N_o nodes to observe being chosen *a priori*, this process will complete the composition of the reduced model by N_s supplementary nodes. These latter must so be selected in order to obtain an optimal response on the observation nodes. The matrix $\boldsymbol{Y}$ is inverted only once. We have seen that each network reduction operation only entails very quick computations. All the $N_\mathrm{o} + 1$ nodes possible models are then built and simulated and an error criterion is computed. This error criterion is computed comparing, for the observation nodes, the responses of the reduced and the detailed models (Eq. (10)).

$$J = \frac{1}{N_o}\frac{1}{t_s}\cdot\int_0^{t_s}\varepsilon(t)\,dt$$

$$\text{with } \varepsilon(t) = \left|T_{\text{detailed}}(t) - T_{\text{reduced}}(t)\right| \tag{10}$$

The input condition are chosen so as to best represent a classical repartition of the sources in the detailed model. We apply an input which is general and rich in frequencies like a step input. The choice of this input maybe adapted to the thermal problem associated with the model but should not be chosen too particular.

The best first supplementary node being chosen by its lowest criterion, the following nodes are computed similarly. Once the N_s supplementary nodes are chosen, a series of iterations allows to stabilise this solution: each of these nodes are successively eliminated to determine again the new last supplementary node. The computation stops when the choice of the N_s nodes stays invariable. A scheme represents this optimisation process in Fig. 6. This strategy is not optimum as not all combinations are tested (this would take about 10^{12} years of computing time for our induction machine implementation), but we can assume that no significant improvement would be added by such a "perfect" combination.

Fig. 7 presents two implementations of this optimisation on reduction of the wall model already mentioned. The presented simulations are in response to a heat step imposed at node 1. In the first example, the observation node is the 1. The improvement brought by a best choice of two other nodes is obvious. In the second example, the observation node is the 129; the improvement is here less important, the

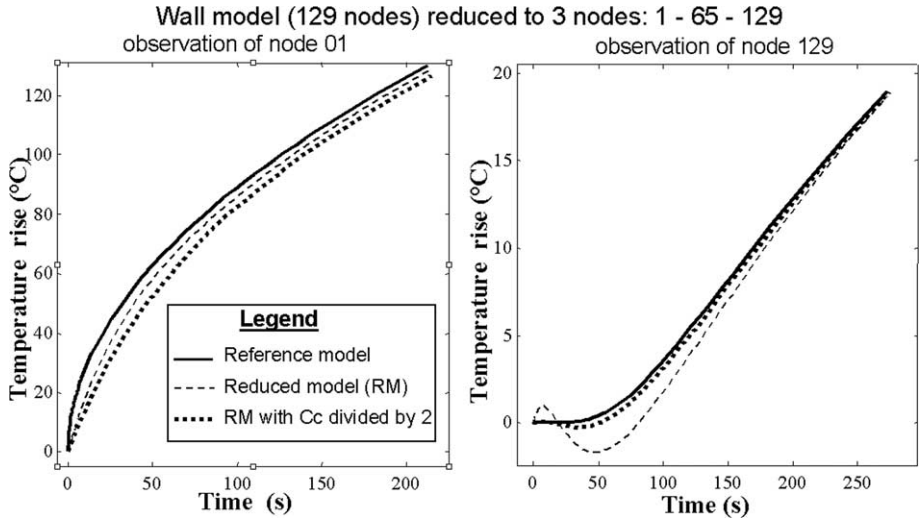


Fig. 5. Influence of the compensation capacitances values.

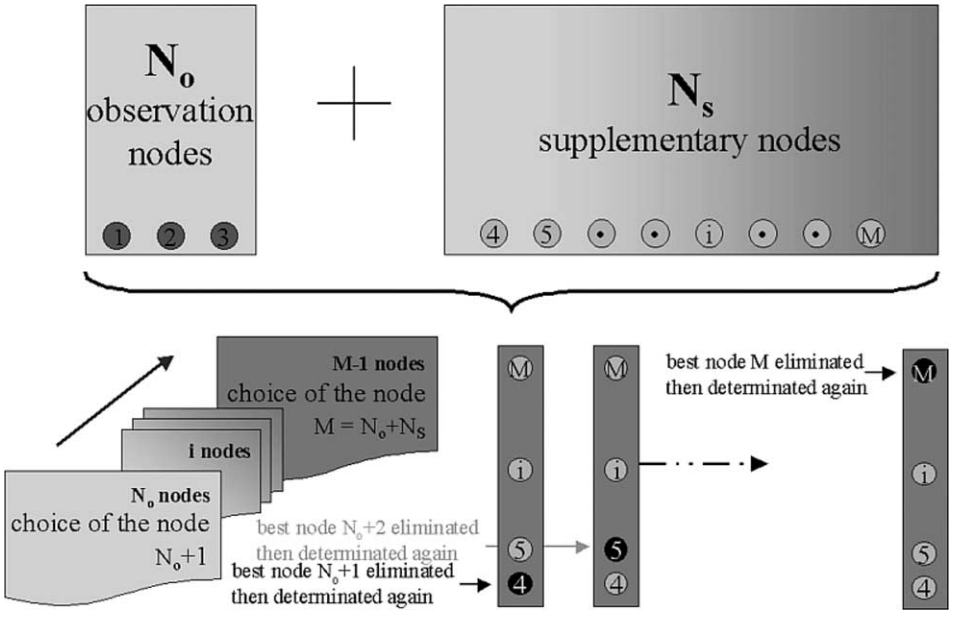


Fig. 6. Diagram of the used process for the supplementary nodes optimised choice.

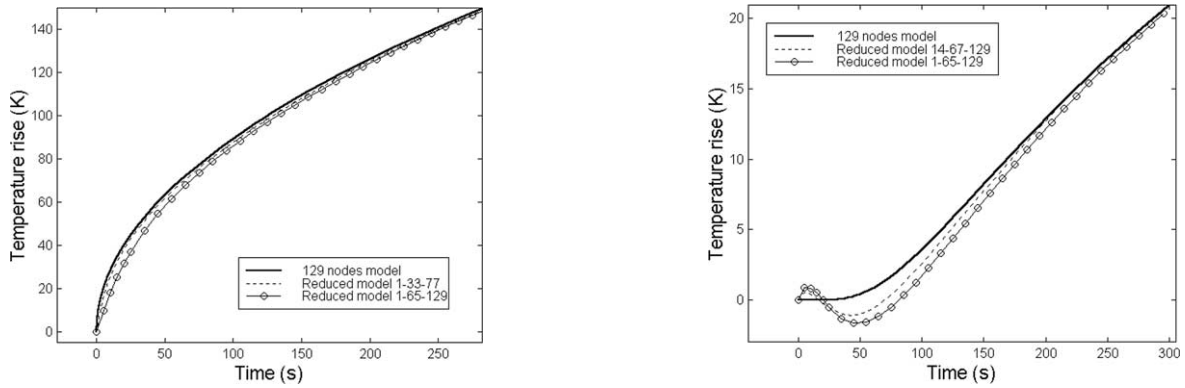


Fig. 7. Responses of optimised 3 nodes reduced models: (a) Node 1; (b) Node 129.

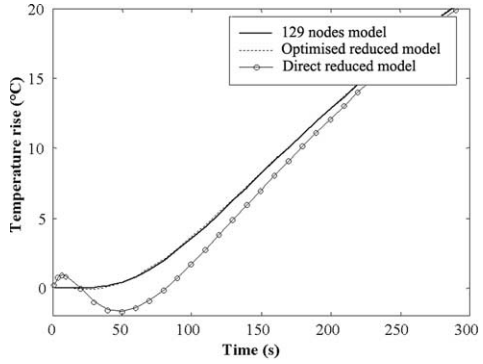


Fig. 8. Response of the reduced model (RM) optimised for the observation of node 129.

oscillation is barely damped, this oscillation is not dependant of the choice of the nodes.

4.2. Optimisation of the compensation capacitances

We mentioned that the imaginary part of the admittance matrix $\boldsymbol{Y}$ contains the terms of heat capacitances. Concerning a reduced model, each of these terms are itself composed of two types of capacitors in parallel whose values are added. One is only present in the diagonal of the detailed model matrix $\boldsymbol{Y}$ and represents the nodal heat capacitance so it has a known physical interpretation and its value should not be modified. The second one represents the compensation capacitance.

In order to optimise the reduced model response, acting on the values of the compensation capacitances, the first operation consists on isolating them in a real matrix $\boldsymbol{CN}$. This matrix is then multiplied term by a matrix **weight** containing weighting coefficients. The obtained matrix $\boldsymbol{CN'}$ is injected again in the reduced model which is simulated and an error criterion is computed with the response of the observation nodes. The simulation conditions are of the same kind as during the previous optimisation. The optimisation process consists in the identification of the matrix **weight** values which minimise the error criterion.

Table 1
Compensation capacitances values of the wall reduced models

	$Cc_{(1-65)}$	$Cc_{(65-129)}$
Raw reduced model (Fig. 6)	$C/12$	$C/12$
Optimised reduced model	$C/26.4$	$C/80.6$

We use, for the resolution, the simplex search algorithm of Nelder and Mead [11] with the *Optimization toolbox* of Matlab. It is necessary to successively start the computation from various initial values as some weighting terms can stay locked to a local minimum. We use initialisations to 1 and to 10^{-3} and to a combination of the first results. It has to be noticed that the identification of these parameters is constrained by the stability of the corresponding models. Each computation is then preceded by a diagonalisation of the state matrix $\boldsymbol{A}$ (Eq. (11)). In order to control the stability of the reduced model, this matrix is verified not to contain any negative eigen value. This operation has an insignificant incidence on computing time.

$$\boldsymbol{A} = -\boldsymbol{C}_{\mathrm{a}}^{-1} \cdot \boldsymbol{G} = -\big(\mathrm{Im}(\boldsymbol{Y})^{-1} \cdot \mathrm{Re}(\boldsymbol{Y})\big) \tag{11}$$

We apply this optimisation to the case of the wall model reduced to the three nodes of Fig. 4, watching the response of the node 129. In this case we have only two compensation capacities values to modify as in this particular case the matrix $\boldsymbol{Y}$ does not include any compensation term. The values of these elements in function of C, the heat capacity of the wall, are given in Table 1. The optimisation of these values allows to obtain the result presented in Fig. 8. The improvement is clear: the response at node 129 of the reduced model is identical to the one of the reference model.

5. Implementation to a complex industrial object

This last point aims to present an implementation of the reduction method on an industrial object. We look into the thermal behaviour of a squirrel cage asynchronous machine of nominal power 5.5 kW. A nodal detailed model of about 1200 nodes is built. The hypothesis of angular symmetry

by section, linked on the stator and rotor notches, allows to only model an elementary sector of the machine. A diagram representing the cutting of the detailed model is presented on Fig. 9. The sources are of four types: stator Joule losses, rotor Joule losses, iron losses (for both stator and rotor) and the mechanical losses mainly situated in the bearing. As this study only concerns the reduction method, we assume that these various losses and their repartition in the machine are well known. The reference simulations used during the optimisation steps takes into account the spatial distribution of the losses corresponding to a classical operation of the machine. Each of the model sources is then a flux step the amplitude of which is the one observed during an operation at nominal power. We choose to keep for observation three nodes supposed to be the hottest spots of the machine: the centre of a rotor squirrel cage bar, the centre of a stator notch (windings) and the centre of the front end windings (the ventilation being in the back side of the machine). Before to study the influence of the reduced model size, we fix the number of nodes to 10.

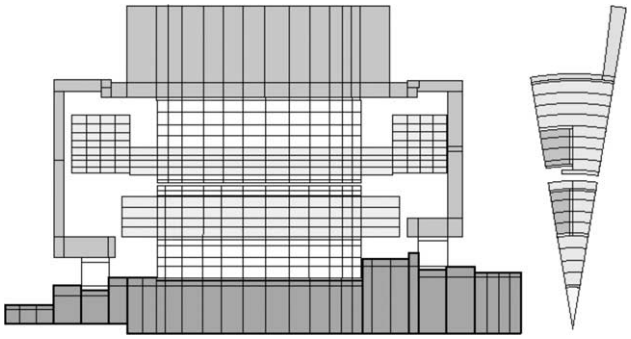


Fig. 9. Diagram of the cutting of the machine's model.

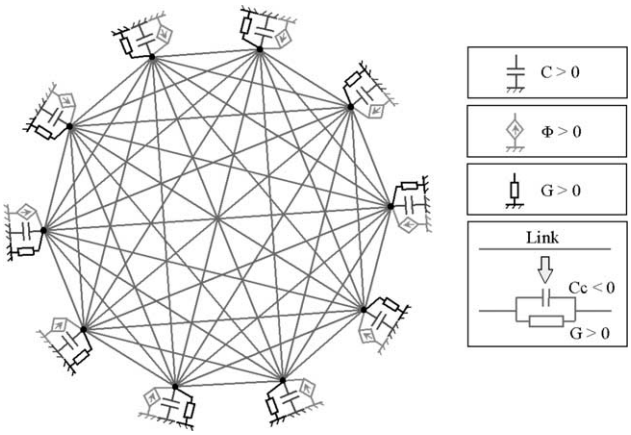


Fig. 10. Polyhedral structure of the motor reduced model.

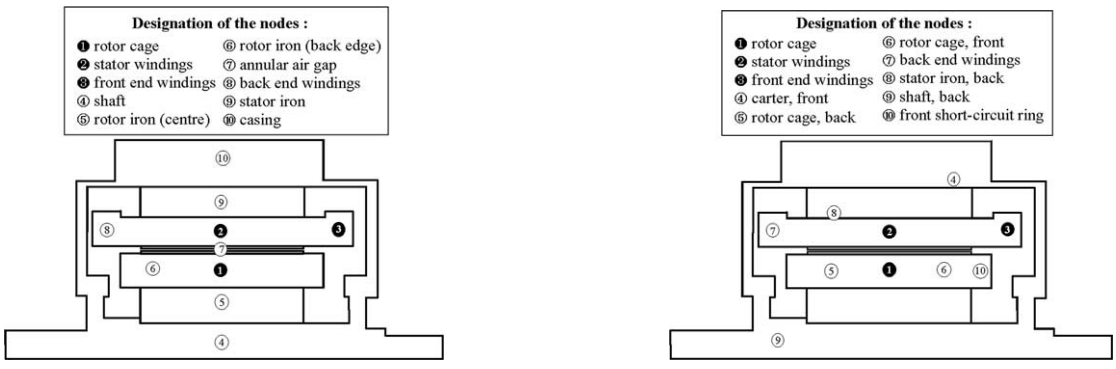
A first reduced model is built from the three observation nodes and 7 supplementary nodes, arbitrarily chosen and without any optimisation step. The obtained reduced model is schematised in Fig. 10 for its polyhedral structure. The reduced admittance matrix is indeed a full matrix and each linkage (conductance or capacitance) between two nodes has a nonzero value. Then there is, between each couple of nodes, a link constituted by a positive conductance and a negative capacitance in parallel.

The optimisation of the choice of the nodes is then applied to the machine model. Fig. 11 presents the situation of the kept nodes for the arbitrary and optimised choices. The nodes of the stator core, shaft and casing are still present but moved backward in the machine model for the two first and forward for the latest. The nodes situated in the annular air gap and the rotor core have been replaced by two supplementary nodes in the cage of the rotor so there is four nodes eventually chosen in that element. 7 nodes out of 10 are situated in a heat dissipation zone, this seems to have a real importance in the conservation of the main information of the model.

We present simulation results in response to a power input which distribution inside the model is described in Table 2. The time variation of the sources is imposed by a weighting factor plotted versus time in Fig. 12. Fig. 13 presents the

Table 2
Distribution of the losses for the presented simulation

Mechanical losses	Stator joule	Rotor joule	Iron losses	
(bearings)	losses	losses	rotor	stator
26 W	533 W	192 W	226 W	
			86 W	140 W



(a) (b)

Fig. 11. Spatial position of the nodes kept for reduction: (a) Arbitrary choice; (b) Optimised choice.

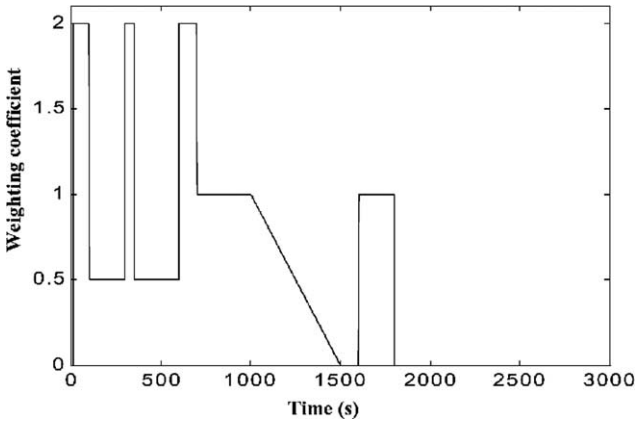


Fig. 12. Time variation of the weighting factor applied to the sources.

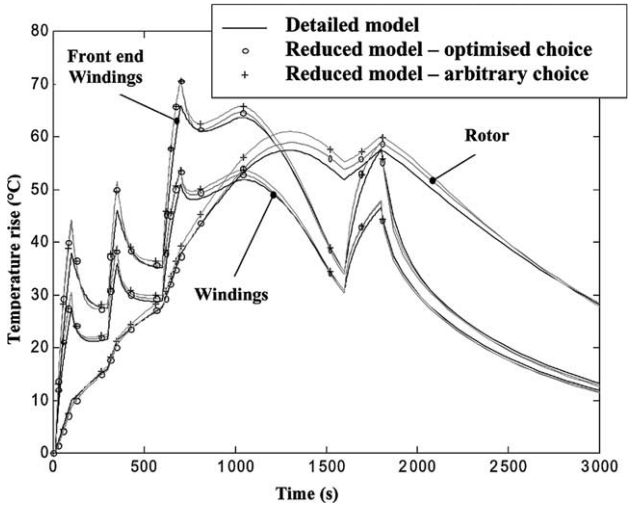


Fig. 13. Compared responses in simulation—arbitrary and optimised reduced model.

simulation results of the detailed, reduced and reduced with optimised choice of the nodes. The corresponding criteria, which is the error summed on the three observed nodes integrated over the simulation time, is almost divided by 2 after this first optimisation step. The value of this error criteria divided by the simulation time and by the number of observed nodes (3 here) is indeed equal to 0.49 K before and 0.27 K after the optimisation. It has to be specifically noted that this criteria is chosen to be calculated from a step input simulation, which is the same used for the optimisation process.

Concerning the number of kept nodes, it logically depends on a compromise between quality and computing time. 10 nodes is here an arbitrary choice, Fig. 14 presents the progress achieved by increasing the order of the reduced model, optimising the choice of the supplementary nodes. One can judge, for each reduced model from 3 to 15 nodes, the error criteria as defined above. Concerning the computing time, the difference between each one is not significant for such small models.

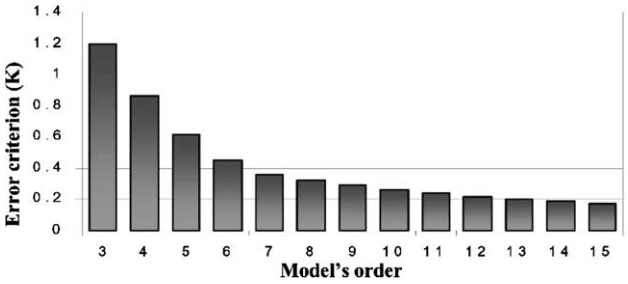


Fig. 14. Improvement of the error criteria in function of the model size.

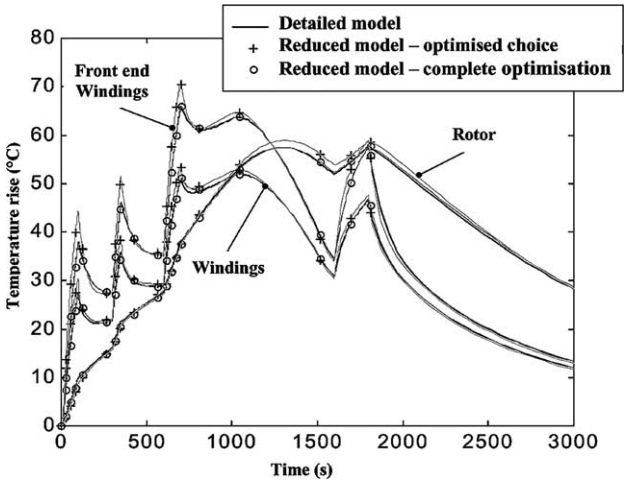


Fig. 15. Compared simulation responses: Detailed model and reduced model before and after the second optimisation step.

The optimisation of the values of compensation capacitances is applied to the new reduced model to improve the response of the three observed nodes. The eventually obtained error criteria is 0.02 K, that is to say a global error 18 times lower than for the non-optimised model. The complete identification of the weighting coefficients matrix for 5 successive initial values represents about 7 hours of computing time for a CeleronII 566 MHz. The simulation results of this model is represented in Fig. 15, compared to the previously discussed one and to the detailed one.

The fitting between the detailed and optimised reduced models is almost perfect. The progress brought by the compensation capacitance optimisation is very noticeable and allows, with 10 nodes, to be as efficient as with 1200 nodes.

Concerning the supplementary nodes temperature responses, it is obvious that there is no reason for it to be improved by the optimisation for they are not monitored in the compensation capacitances optimisation process. It is although observed in our example each node response has improved except the one situated on the shaft (node 9 on Fig. 11), and their quality is close to the observed nodes. One can then doubt on the utility of the optimisation of the choice of these nodes. Computations have been done to apply the second step of optimisation on various arbitrarily composed reduced models (containing the same three observed nodes).

The error criteria appears to be systematically greater than for the completely optimised reduced model. The two steps of optimisation then seem to be complementary.

6. Conclusion

A model reduction method based on the Kirchhoff's network theory has been described and first implemented to a simple and 1 D model of a wall. It has been then applied to the model of an induction machine of 5.5 kW. This method allows to reduce the topology of a network composed of conductances, capacitances and flux inputs and constituting a model of heat diffusion inside a complex object. A few nodes are kept during a manipulation of the admittance and heat sources matrices to obtain a new network of similar structure. Some new elements, the compensation capacitances, then appear in the reduced topology and allow to obtain good responses in transient. Two optimisation steps are completing the reduction process in optimising its nodal composition as well as the values of the compensation capacitances. The simulation results show a very good fidelity of the reduced model compared to the original detailed model a hundred times larger.

If developments concerning model reduction seem generally oriented towards modal analysis, this method shows all its interest and originality in its simplicity to carry out for a very accurate result. It then opens important perspectives of industrial implementations as it can be applied by any engineer. Perspectives of application on determination of the modelled object properties or sources using inverse techniques is also to consider.

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